Exponential Growth and Inhibited Growth

a) The differential equation $\dfrac{dy}{dx} = ry$ describes a situation in which a population size y increases at a rate proportional to its size. Use separation of variables to find a solution to this equation.

b) The differential equation $\dfrac{dy}{dx} = ry(s-y)$ $(s > 0)$ describes change in a population which tends toward a fixed size s. For example, this might describe a population in which food or space is limited. Use separation of variables and the fact that $\displaystyle\int \frac{dy}{y(s-y)} = \frac{1}{s}\ln\left|\frac{y}{s-y}\right| + c$ to find a solution to this equation.

5/8/25

Exponential Growth and Inhibited Growth

a) The differential equation $\frac{dy}{dx} = ry$ describes a situation in which a population size y increases at a rate proportional to its size. Use separation of variables to find a solution to this equation.

b) The differential equation $\frac{dy}{dx} = ry(s-y)$ $(s > 0)$ describes change in a population which tends toward a fixed size s. For example, this might describe a population in which food or space is limited. Use separation of variables and the fact that $\int \frac{dy}{y(s-y)} = \frac{1}{s}\ln\left|\frac{y}{s-y}\right| + c$ to find a solution to this equation.

a)

$$\frac{dy}{dx} = ry$$

$$\int \frac{1}{y}\,dy = \int r\,dx$$

$$\ln|y| + C_1 = rx$$

$$|y| = e^{rx} \cdot e^{c_1}$$

$$y = Ce^{rx}$$

b)

$$\frac{dy}{dx} = ry(s-y), \quad s > 0$$

$$\int \frac{1}{y(s-y)}\,dy = \int r\,dx$$

$$\Rightarrow \frac{1}{s}\ln\left|\frac{y}{s-y}\right| + C = rx$$

$$\ln\left|\frac{y}{s-y}\right| = s(rx - C)$$

$$\frac{y}{s-y} = \pm\frac{e^{srx}}{e^{sc}}$$

$$y = Ae^{srx}(s-y)$$

$$(1 + Ae^{srx})y = Ae^{srx}s$$

$$y = \frac{Ae^{srx}s}{1 + Ae^{srx}}$$